

# Optimization Algorithms as Quantum Performance Benchmarks

Pratik Sathe

University of California at Los Angeles (UCLA),  
& Universities Space Research Association (USRA)

March 06, 2023



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Collaboration with Tom Lubinski, Carleton Coeffrin, Joshua Apanavicius, Catherin McGeoch and David Bernal.

# Background and Motivation

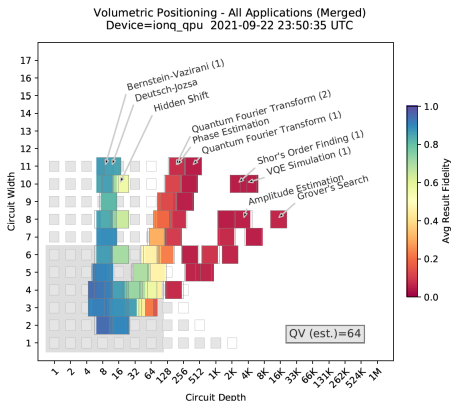
- Component-level benchmarking is valuable, but less informative for the user.
- End-User perspective: Will I be able to solve my problem using a quantum computer?
- Hence, QED-C's approach <sup>1</sup>:
  - Versatile, accessible benchmarking suite.
  - How good an answer does the hardware give?
  - Use out-of-the-box software capabilities.
  - Run algorithms on hardware, and present insightful visualizations.
  - Plug and play

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<sup>1</sup><https://github.com/SRI-International/QC-App-Oriented-Benchmarks>

# Volumetric Positioning

- Sub-routine and algorithm benchmarking<sup>2</sup>:
  - Quantum Fourier Transform, Phase Estimation, etc.
  - Grover's Algorithm, Hamiltonian Simulation, etc.



*Volumetric Positioning* plot obtained  
from the first installment of the suite

<sup>2</sup>Lubinski, Thomas, et al. "Application-oriented performance benchmarks for quantum computing."  
<https://arxiv.org/abs/2110.03137>

# Performance for Combinatorial Optimization Problems

- Example: Find  $s_1, \dots, s_N$  (each being 0 or 1) which minimize<sup>3</sup>

$$\text{Cost function } H := \sum_{i,j} J_{ij} s_i s_j + \sum_i h_i s_i$$

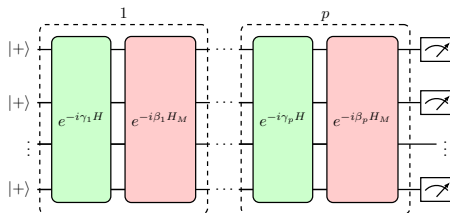
- NP-hard
- Digital/Gate model QPUs: Quantum Approximate Optimization Algorithm<sup>4</sup>: A hybrid quantum-classical approach offering potential speed-up.
- Analog QPUs: Quantum Annealing

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<sup>3</sup>aka QUBO problems (Quadratic Unconstrained Binary Optimization).

<sup>4</sup>Farhi, Edward, Jeffrey Goldstone, and Sam Gutmann. "A quantum approximate optimization algorithm." arXiv preprint arXiv:1411.4028 (2014).

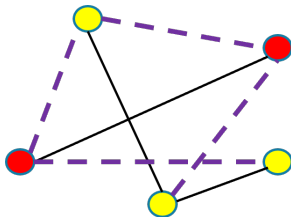
# QAOA: Quantum Approximate Optimization Algorithm



**Figure:** QAOA Circuit: Each circuit corresponds to a probability distribution over 'solution space'.

- 1 Choose initial parameters  $\beta_1, \dots, \beta_p, \gamma_1, \dots, \gamma_p$ .
- 2 Implement circuit many times; obtain  $\langle H \rangle$ .
- 3 Classical minimizer routine updates angles that result in smaller  $C \equiv \langle H \rangle$ .
- 4 If not converged, go back to (2).

# Choice of Problem: The MaxCut Problem



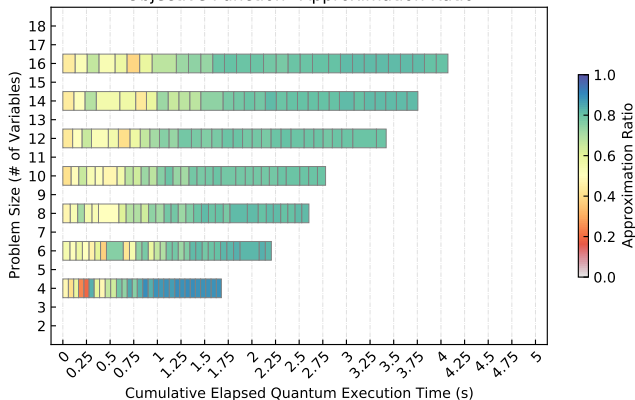
- **Given:** Graph  $G$  with vertices  $V$  and edges  $E$ .
- A '**cut**' is a division of the vertices into two groups.
- The '**size**' of a cut is the number of edges that now connect vertices from different groups.
- **Objective:** Find the cut with highest size, i.e. the 'Max Cut'.

QAOA Formulation: Find ground state of:

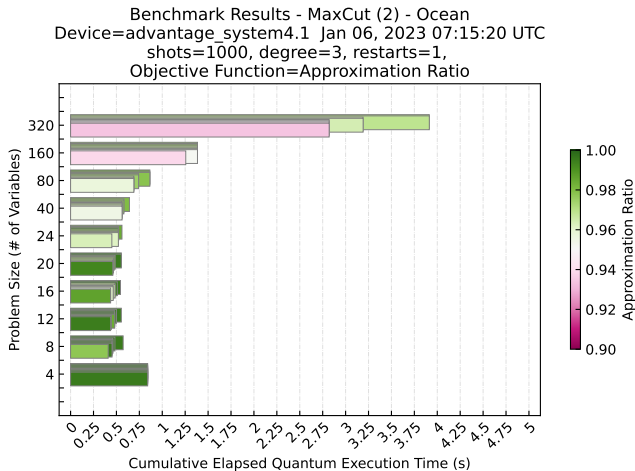
$$H = -\frac{1}{2} \sum_{\langle j,k \rangle \in E} (1 - Z_j Z_k),$$

# Area Plots: Time Evolution of Quality

Benchmark Results - MaxCut (2) - Qiskit  
Device=qasm\_simulator Jan 06, 2023 08:00:46 UTC  
shots=1000, rounds=2, degree=3, restarts=1, fixed\_angles=False,  
Objective Function=Approximation Ratio



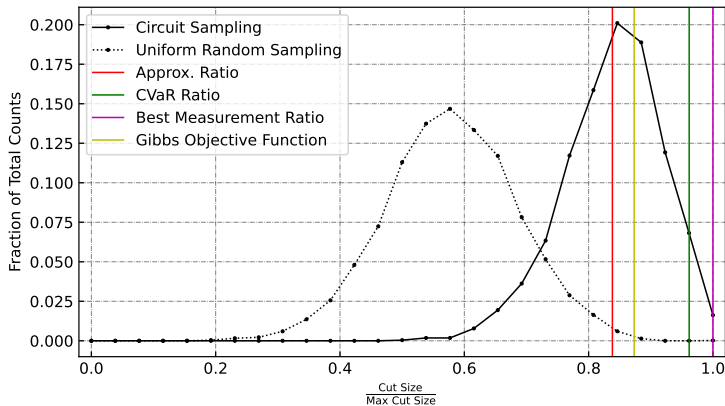
# Area Plots: Time Evolution of Quality





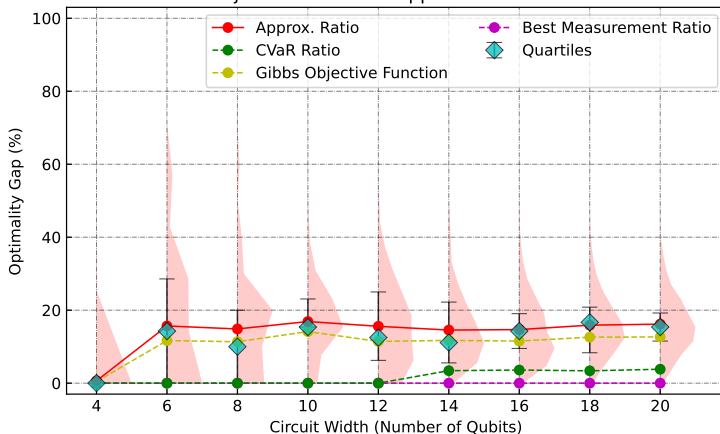
# Distribution of Cut Sizes

Empirical Distribution of Cut Sizes - MaxCut-(2)  
Device=qasm\_simulator Oct 10, 2022 22:50:49 UTC  
shots=5000, rounds=2, degree=3, restarts=1,  
Objective Function=Approximation Ratio  
width=20



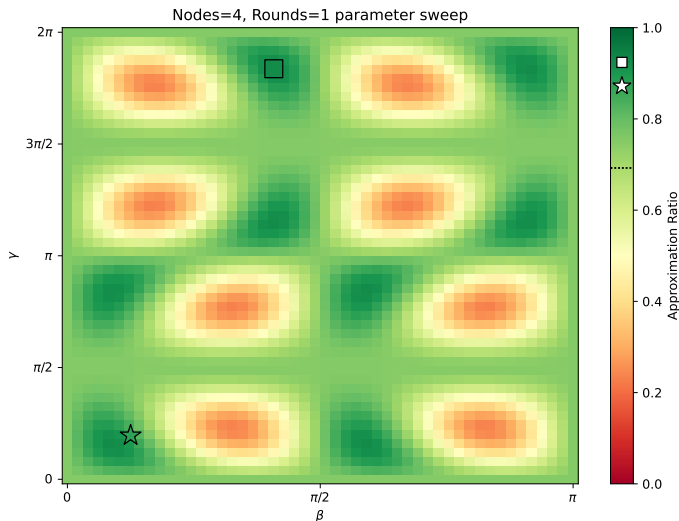
# All Metrics and Distribution all Widths

Benchmark Results - MaxCut (2) - Qiskit  
Device=qasm\_simulator Oct 10, 2022 22:50:46 UTC  
shots=5000, rounds=2, degree=3, restarts=1,  
Objective Function=Approximation Ratio



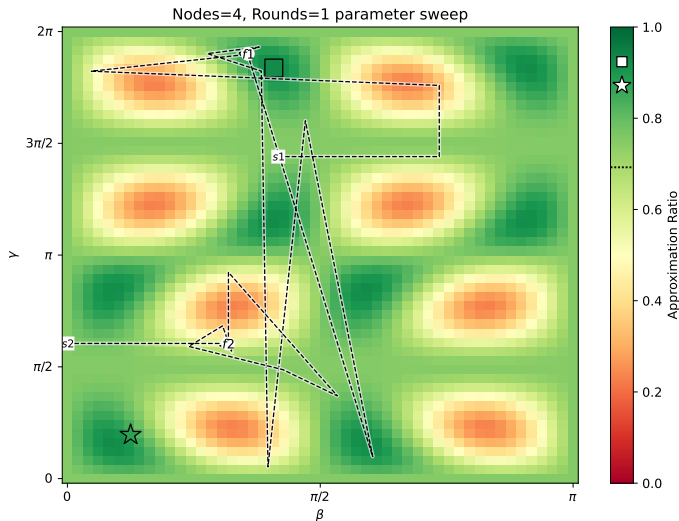
# Cost Function Landscape

Created using state-vector simulator  $\beta \in (0, \pi]$  and  $\gamma \in (0, 2\pi]$



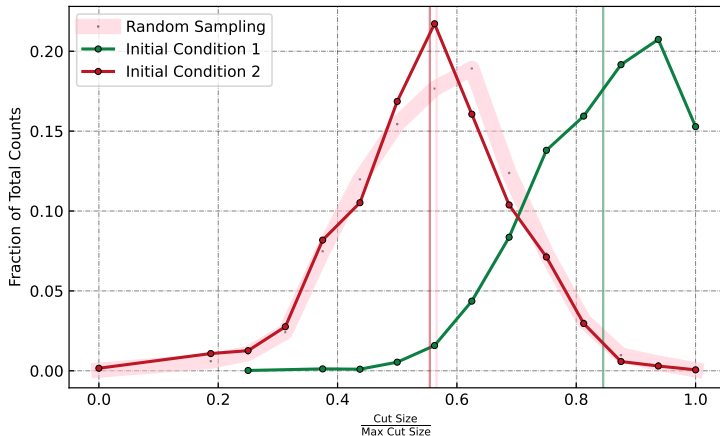
# Parameter Trajectories

QAOA with 30 rounds, random initial conditions



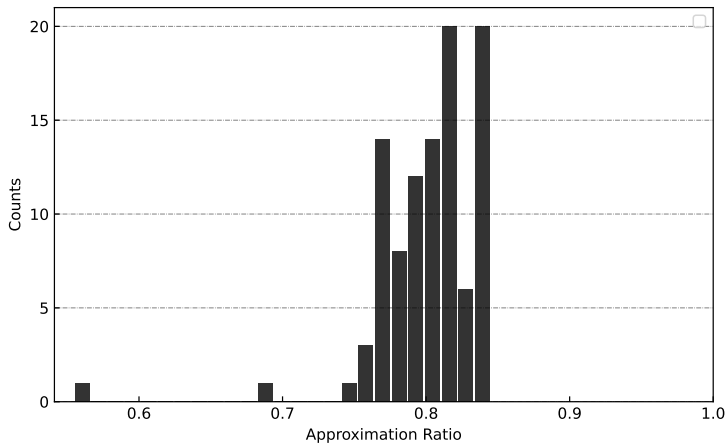
# Initial Conditions Affect Quality Significantly

Empirical Distribution of cut sizes  
Device=qasm\_simulator Sep 09, 2022 22:49:44 UTC  
shots=5000, width=12, degree=3, restarts=100



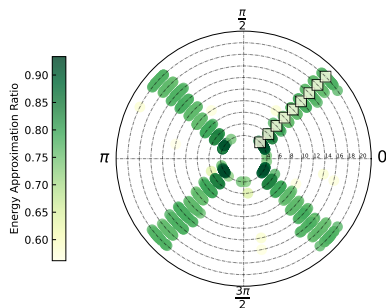
# Histogram of Approximation Ratios with 100 restarts

Histogram of Approximation Ratios  
Device=qasm\_simulator Sep 09, 2022 22:49:48 UTC  
shots=5000, width=12, degree=3, restarts=100

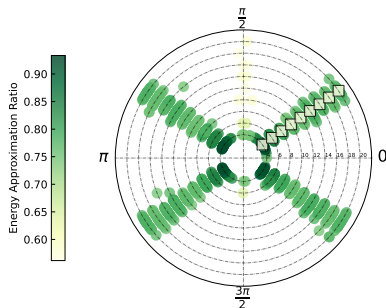


# Radar Plots (Rounds=1)

Benchmark Results - MaxCut (2) - Qiskit  
rounds=1, Objective Function=Approximation Ratio  
 $2\beta_1$



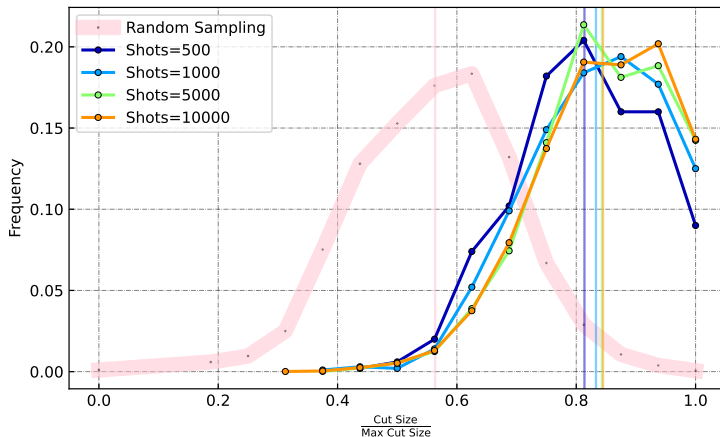
Benchmark Results - MaxCut (2) - Qiskit  
rounds=1, Objective Function=Approximation Ratio  
 $\gamma_1$



Shots = 1000

# Effect of Number of Shots

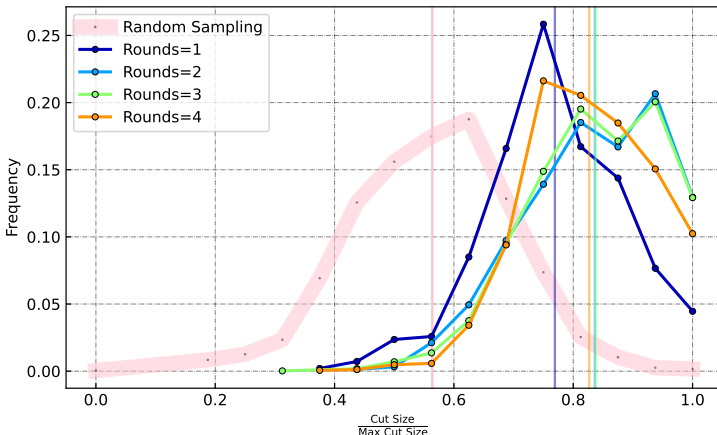
Empirical Distribution of cut sizes  
Device=qasm\_simulator Oct 11, 2022 16:32:50 UTC  
width=12, degree=3, restarts=1





# Effect of Rounds

Empirical Distribution of cut sizes  
Device=qasm\_simulator Oct 11, 2022 06:03:44 UTC  
width=12, degree=3, restarts=1



Same starting angles (all 1's).

# Conclusion and future work

- Open-source benchmarking framework
  - Works on different hardware modalities
  - End-users as well as researchers in mind
  - Also useful to understand effects of parameter choices.
- Details presented in manuscript<sup>5</sup>.
- Extend framework:
  - Apply to other iterative hybrid algorithms, such as VQE.
  - Enhancements with 3<sup>rd</sup> party compiler optimization tools.

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<sup>5</sup>Thomas Lubinski, Carleton Coffrin, Catherine McGeoch, Pratik Sathe, Joshua Apanavicius, and David E. Bernal Neira. "Optimization Applications as Quantum Performance Benchmarks." arXiv preprint [arXiv:2302.02278](https://arxiv.org/abs/2302.02278) (2023).

Thank you!